Minimizing valve seat leakage

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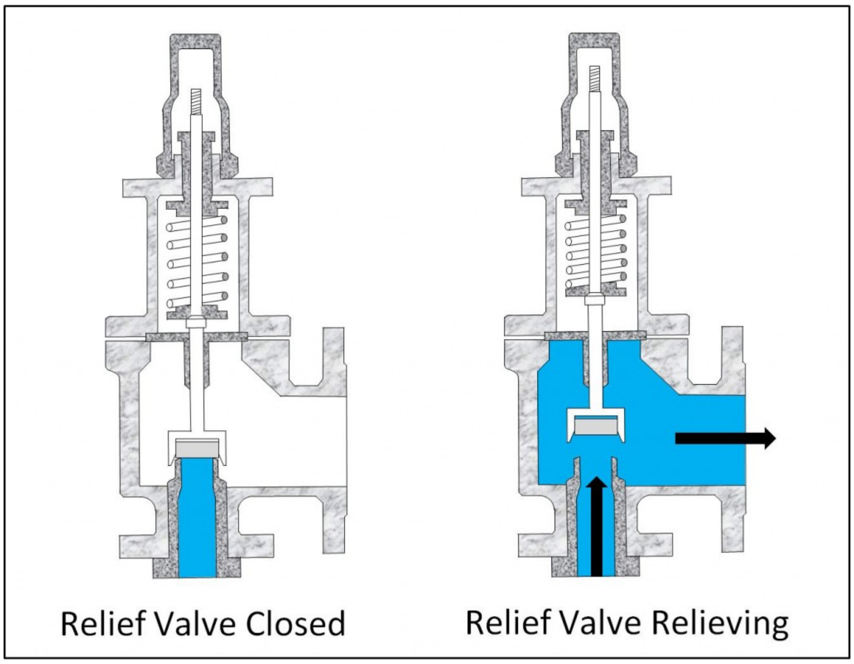
Abstract

A mathematical model has been developed to simulate pressure relief valve seat leakage und conditions of angular and misaligne loading. The model incorporates loading parameters as well as relevant geometric properties of valve components. The model can be implemented to establish tolerable levels of misaligned and angular loading for a valve with a specified set of design parameters. The operating limitations of a 3/4 in. inlet by 1 in. outlet Crosby JMBL valve were evaluated using the model. The limit of angular loading was shown to theoretically be 1.255 degress, and the tolerable level of misalignment was shown to be 0.0167 inches.

The disc radius and the distance from the point of load application to the disc sealing surface were varied independently to link valve performance trends to these parameter values. Increasing the disc radius was shown to minimize seat leakage under a specified set of loading conditions. Decreasing the distance between load application and the disc sealing surface was shown to further reduce seat leakage. Concerns about implementing these design modifications are discussed, and appropriate recommendations for design and testing are submitted.



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1 Introduction

1.1 Functions and Applications of Pressure Relief Vavles

A pressure relief valve is an automatic pressure relieving device that is mounted on a pressurized system to relieve the system of excess pressure when abnormal operating conditions cause the pressure to exceed a set limit. The primary purpose of any pressure relief device is the protection of life and property by preventing dangerous overpressure in vessels, lines, or systems. Some common applications of pressure relief valves are chemical plants, nuclear plants, and any other facility using pressurized gases of liquids. Since the valve is a safety mechanism, the reliability an simplicity of the design are of upmost importance. Codes and standards regulating materials, design parameters, and ranges of application have been developed to ensure that safe standards are upheld in both design and implementation of pressure relief valves.

1.2 Valve Operation

A basic knowledge of the structure and function of a pressure relief valve is essential to fully understand valve design and performance issues. A pressure relief valve must open at a predetermined set pressure, flow at a rated capacity, and reseal tightly when the system pressure has returned to a safe level. The most commnon pressure relief valve consists of an inlet nozzle mounted to a pressurized system, a spring-loaded disk held against the nozzle to prevent flow under normal operating conditions, and an external housing to contain the internal workings of the valve (Figure 1-1). When the load on the underside of the disc overcomes the mechanical load of the spring on the disc, the valve opens and releases the excess pressure at the outlet. Valves are used for both liquid and gas applications with only slight modifications in valve structure. Although this study will use a liquid operating valve as its base case, the problems and solutions developed will be equally applicable to the gas operation case.

1.3 Crosby Valve and Gage Company

Crosby Valve and Gage Company is a leading manufacturer of pressure relief valves. Crosby has been in business for 120 years, producing a variety of pressure and safety relief instruments for national and international markets. Crosby is a market leader in pressure relief valves, holding a 22% share of the market. Roughly 65% of sales are to process industries, primarily chemical plants. The remaining 35% of sales is largely accounted for by power industries.

Crosby's worldwide operations are headquartered in Wrentham, Massachusetts, where the primary manufacturing plant is located. Significant manufacturing is also performed at facilities in England and Canada. Manufacturing is conducted almost exclusively in a job shop environment, with machine setups varying to accommodate orders as they arrive. Crosby also has complete high pressure testing facilities for steam, air, and liquid applications.

In an effort to maintain their competitive edge in the market, Crosby is currently investigating alternate designs and manufacturing systems to improve valve performance and reduce production costs.

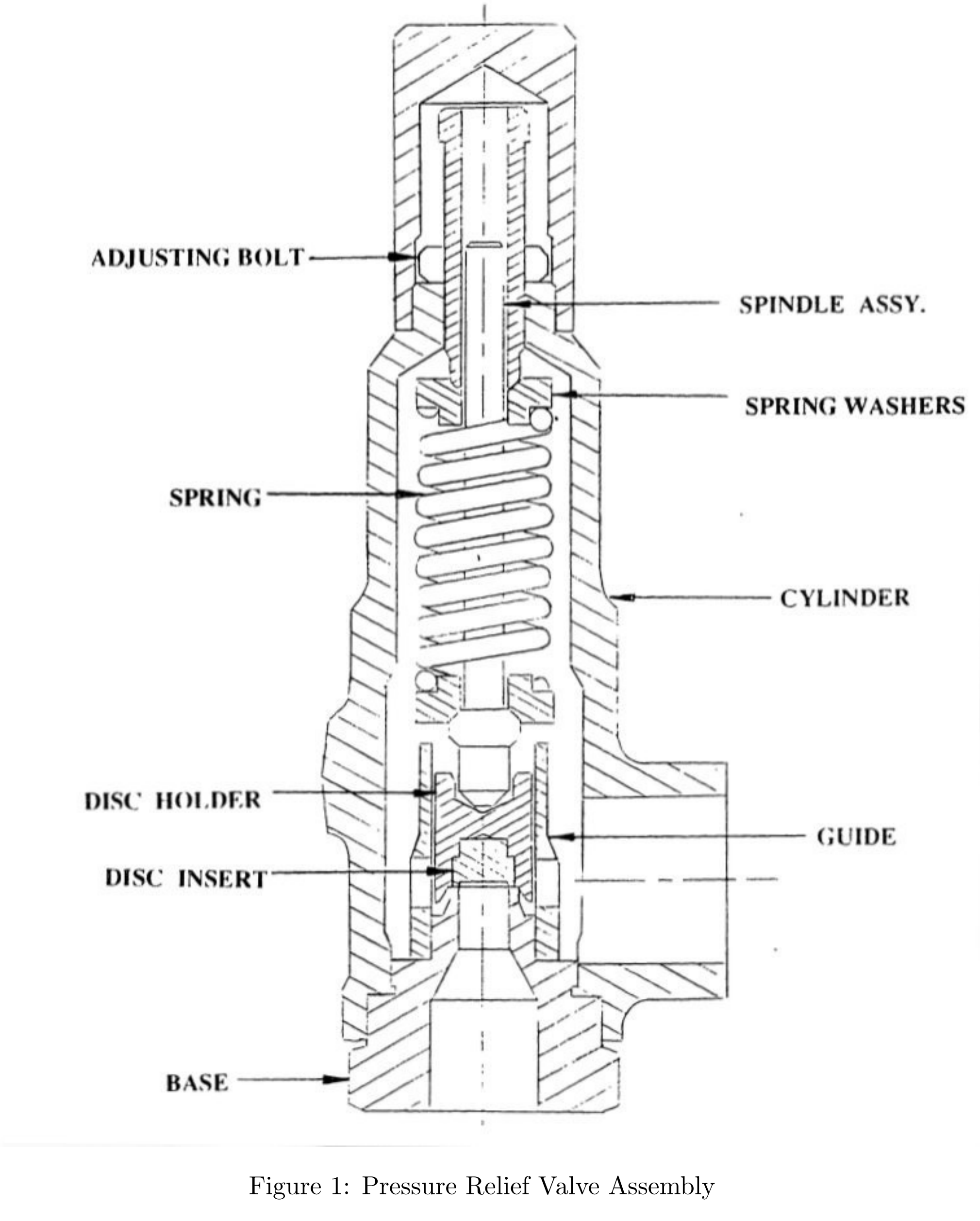


Figure 1: Pressure Relief Valve Assembly

1.4 Project Overview

This project entails working in conjunction with Crosby Valve and Gage Company towards improving pressure relief valve performance under conditions of misaligned and angular loading. It is the goal of this project to evaluate the effects of misapplied loading on valve seat leakage and develop component designs which minimize the leakage problem.

An introduction to the causes of valve leakage will be presented, followed by alternative design approaches for minimizing leakage. A mathematical model of pressure relief valve seat leakage will be developed, incorporating relevant loading parameters and geometric properties of valve components. Implementation of this model will define the tolerable limits of misaligned and angular loading for a given set of valve design parameters. The model will also evaluate the effects o varying geometric properties of valve components, indicating performance trends and optimal values for component dimensions. The model and optimization data will provide Crosby with a theoretical basis for pressure relief valve component designs which eliminate leakage problems.

2 Valve Leakage

2.1 Valve Leakage

An important consideration in the design, manufacture, and implementation of a pressure relief valve is maintaining a tight pressure seal between the disc and inlet nozzle in order to minimize seat leakage. This seal tightness can be achieved in part by close control of part alignment, optically flat seating surfaces, and careful sizing and selection of materials for individual applications. However, if any one or more of these conditions is not adequately met, it becomes very likely that the valve will leak.

Seat leakage is a problem that must be seriously addressed by both valve manufacturers and customers. Industry standards require that all valves must pass a leak test before they can be shipped out for industrial use. Valves that fail the leak testing become very costly to the manufacturer since the leak test is a final test, and there is no way to determine whether a valve will leak at any stage prior to final assembly and testing. If a valve leaks, it must be returned to manufacturing for adjustments or further machining. The valve must then be reassembled and leak tested once again. Since assembly and testing currently accounts for almost one third of Crosby's total production time, a large number of failed valves will result in inefficient production. costing the company time and money that could be dedicated to other tasks.

Even valves that meet requirements at the factory often require maintenance immediately or shortly after installation in a plant. Suppliers have no control over conditions to which the valve may be subjected after shipment. Pressure relief valves are required to remain on systems for prolonged periods of timne under widely varying operating conditions. The effects of thermal distortion on the valve seat, corrosive fluids, and cycling of pressure and temperature all contribute to reducing the integrity of the valve seal. Even the smallest impurities or particles lodged between the guiding surfaces of the valve will gall these surfaces and cause the valve to remain partially or fully open.

It is therefore of utmost importance that a design contribute to minimizing leakage since there are so many operating conditions that contribute to leakage. It is nearly impossible to achieve and maintain zero leakage in a pressure relief valve. A study conducted by the National Aeronautics and Space Administration states that a metal-to-metal valve seat could be made to obtain a leakage rate of less than x 10 ' 3 cc/sec of helium. except that this valve could maintain this low level of leakage for one closing cycle only. For these reasons, allowable seat leakage limits for pressure relief valves have been established. However, these limits are very stringent since the valve is a safety device.

2.2 Pressure Relief Valve Seat Tightness Standards

Most pressure relief valves are built to meet the commercial metal-to-metal seat tightness requirements of API(American Petroleum Institute) Standard 527, which establishes allowable leakage rates for the valves. API Standard 527 specifies apparatus and procedures that should be used for testing seat tightness. It also establishes an allowable leakage rate depending on the size and type of valve.

A typical test arrangement for measuring seat leakage as specified is illustrated in Figure 2-1. The valve shall be mounted vertically to an air receiver, using air at atmospheric temperature as the pressurized medium. Leakage measurements shall be taken from a 5/16 inch outer diameter tubing with .035 inch wall thickness at the valve outlet. The tube end shall be cut smooth and be parallel to and 1/2 inch below the water surface. A leakage rate, measured in bubbles per minute, is determined with the pressure at the valve inlet held at 90% of the set pressure. This pressure is to be maintained for at least one minute prior to counting the bubbles for valve inlet sizes up to 2 inches diameter (this time will increase for larger valves). Prior to testing for seat leakage, all secondary openings and seals should be checked with a soap film for leakage to assure that any leaking fluid escapes through the tube at the outlet. The leakage rates should not exceed the values listed in Table 2-I.

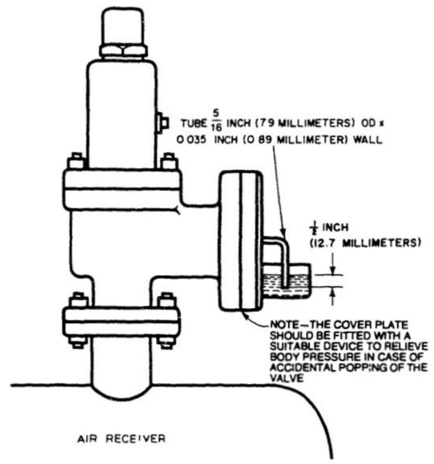


Figure 2: Test Apparatus for Seat Tightness

2.3 Causes of Valve Seat Leakage

Seat leakage in pressure relief valves is due largely to angular and misaligned loading on the valve seat. The effects of this loading are a reduced vertical force component and an unequal force distribution accross the seat of the valve. This nonuniform force distribution generates a moment on the disc. If this applied moment becomes large enough to overcome the sealing pressure on the seat, the disc will begin to lift and the valve will leak.

Failure to maintain close alignment of the valve parts during assembly and operation is a major cause of angular and misaligned loading. Some major contributors to part misalignment are the following:

- sloppy threading - Sloppy threading is one factor that affects part alignment. There are several locations on the valve where cylindrical parts are threaded together. The precision of this threading is crucial for maintaining alignment of the parts along the vertical axis of the valve.
- loose tolerances - Clearances between moving parts and guiding surfaces must be kept large enough to prevent sticking, especially under conditions of thermnal expansion. However, these necessary clearances do allow some lateral movement and misalignment of the guided parts.
- spring buckling - A spring subjected to too large a load or having a large length to diameter ratio may buckle and result in angular or misaligned load application.

2.4 Design Alternatives for Minimizing Seat Leakage

There are two design approaches for minimizing seat leakage in pressure relief valves. Design efforts can be focused on eliminating the misapplied loading conditions. Crosby is currently performing some studies to this effect, with an emphasis on spring design and selection.

An alternate approach which will be taken in this project is to assume the existence of misapplied loading conditions and develop a design which is least affected by these conditions. A mathematical model will be used to simulate valve leakage and establish tolerable limits for angular and misaligned loading for a given set of valve design parameters. Relevant geometric properties of valve components will then be varied around base case values, and optimal values for these parameters can be selected and implemented in future designs.

3 Theoretical Model of Pressure Relief Valve Seat Leakage

3.1 Applications and Scope of Model

In this chapter a theoretical model of pressure relief valve seat leakage will be developed for the purpose of identifying and optimizing relevant design parameters for minimizing seat leakage. An accurate mathematical model of pressure relief valve seat leakage can be a very useful tool for evaluating potential designs and optinizing design parameters. It will make it possible to identify the optimal value of design parameters or combinations of parameters which will yield minimum leakage under specified loading conditions. This model will not address the sources of misapplied or offcentered loading, but will, however, establish acceptable limits for angular and misaligned loading.

This model is not intended to stand alone, but rather to serve as a design tool to be applied in conjunction with existing knowledge and practices. Implementation of such a model can save considerable time in design, production, and testing of valve prototypes by providing a theoretical basis for valve design which minimizes the leakage problem.

3.2 Assumptions

It is necessary to first delineate the assumptions that the model will be operating under. These assumptions are not intended to limit the application of the model or introduce deviations from the actual circumstances, but serve to make simplifications and specifications where necessary in an effort to generate meaningful results that have a clearly defined range of applications. Following is a list of assumptions that were made in developing the model along with a brief justification of each:

- simplified seating geometry - The actual geometry of the disc holder and disc insert have been simplified somewhat to facilitate the use of a free body diagram.
- distributed pressure force of system replaced by a single resultant force acting on the geometric center of the disc insert - The resultant force will produce the same overall forces and moments as the original distribution.
- angle of applied load, ϕ, remains constant during leakage - At the instant of initial leaking, the seat lift is negligibly small. This differential change in position will result in negligibly small changes in the angle of the applied load.
- seating surfaces deformable - This is a necessary and appropriate assumption to make. 316 Stainless Steel will deform under the operating conditions of a pressure relief valve. If the surfaces were treated as rigid bodies, then a pressure seal would not be able to form. It is also necessary to assume no irregularities in surface finish in order to form and maintain a good pressure seal.
- equilibrium exists at the time of leak initiation - At the point of leak initiation, the system must be in static equilibrium. As long as the integrity of the seal is maintained and the disc has not yet lifted, the forces and moments acting on the disc and disc holder must be balanced. As the loading along the seal becomes imbalanced, the valve will begin to leak and this equilibrium condition will be lost.
- force distribution along seal is linear - At leak initiation, the sealing pressure at the point of leakage must be exactly zero. This sealing pressure increases linearly accross the disc to a maximum value at the point on the disc diametrically opposite the leakage point. This pressure will increase linearly, assuming no irregularities in fluid flow or surface finish. Deviations from this linearity will most likely be small enough to be ignored.

- the effects of all factors contributing to misaligned or offcentered loading can be lumped - This is acceptable practice for achieving the goals of this model. This model does not focus on the causes of loading but rather on the leakage effects of this loading.

3.3 Theoretical Analysis

The purpose of this model is to determine the pressure at which a pressure relief valve will leak given a specified set of design parameters, dimensions, and loading conditions. It is first necessary to evaluate all forces and moments acting on the disc. These forces and moments will be evaluated in the equilibrium condition at the instant of leak initiation. To satisfy the condition of static equilibrium, forces and moments must be balanced in all directions. A free body diagram depicting a somewhat simplified disc with the forces acting on it is illustrated in Figure 3-1. The coordinate system as indicated in the figure defines the y direction along the vertical axis of the valve and the x direction in the plane of the sealing surface. The z direction is defined to be perpendicular to the x and y axes. The load is applied completely in the..x-y plane, and moments will be taken about the z axis.

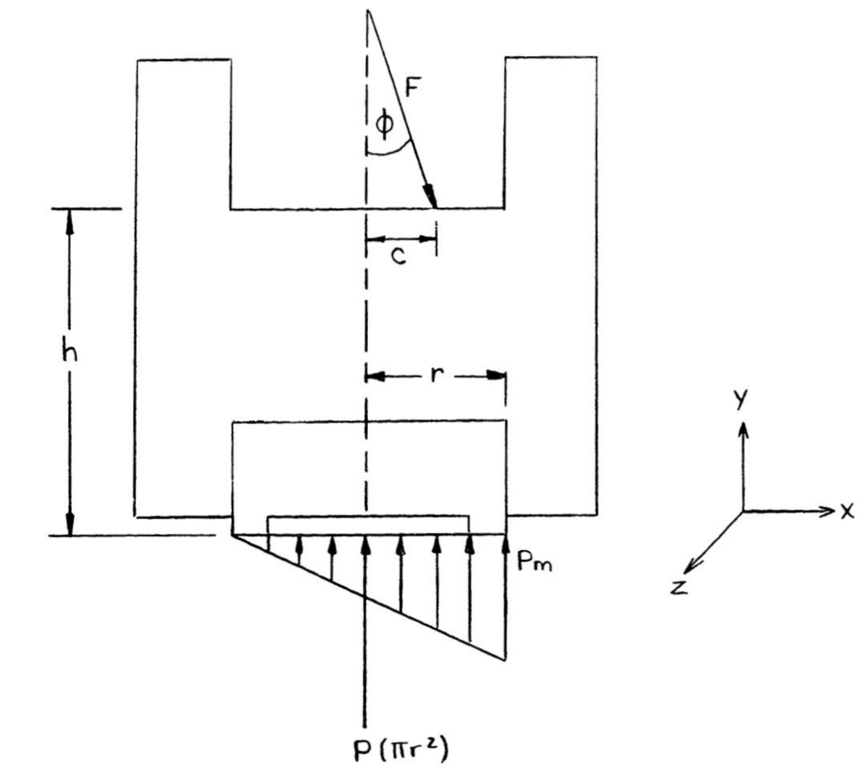


Figure 3: Free Body Diagram of Disc

3.3.1 Equilibrium Force Balance

There are two forces that act on the disc in the x-direction. There is a component of the applied load, F, in the x-direction equal to Fsint, where ϕ is the angle of applied load (vertical = 0 degrees). This force component is balanced by the frictional forces acting between the disc insert and the seating surface of the nozzle (Equation 1).

$$F \cdot sin\ \phi = F_f \tag{1}$$

The tight clearances of the guide also serve to restrict any lateral movement of the disc.

The force balance in the y direction is more complicated since it must account for the component of the applied load, the pressure force of the system, and the distributed pressure force along the sealing surface of the disc. The component of the applied load in the y direction is Fcost, where once again t is the angle of the applied load. The pressure force of the system is treated as one resultant force acting on the center of the disc. This resultant force will produce the same overall forces and moments as the original distribution. This resultant force will have a magnitude equal to the product of the system pressure, P, and the area over which it is applied. Evaluating the distributed pressure force along the sealing surface of the disc is more complicated since the sealing pressure varies from zero at the point of leak initiation to a maximum value at the point directly opposite the leakage point. Figure 3-2 isolates the forces acting on a differential area element of the disc. This pressure force must then be integrated around the complete sealing surface of the disc to conrectly evaluate the distribution (Equation 2).

$$2\int_0^{\pi} \frac{r + cos\ \Theta}{2r} P_m(r\Delta r)d\Theta \tag{2}$$

Evaluating this integral yields the following expression for the force acting on the sealing surface of the disc:

$$\Pi r \Delta r P_m \tag{3}$$

The complete force balance in the y-direction can therefore be expressed in the following form:

$$F \cdot cos\ \Phi = \Pi r \Delta r P_m + \Pi r^2 P \tag{4}$$

A more complete derivation and explanation of the components of this equation can be found in Appendix A.

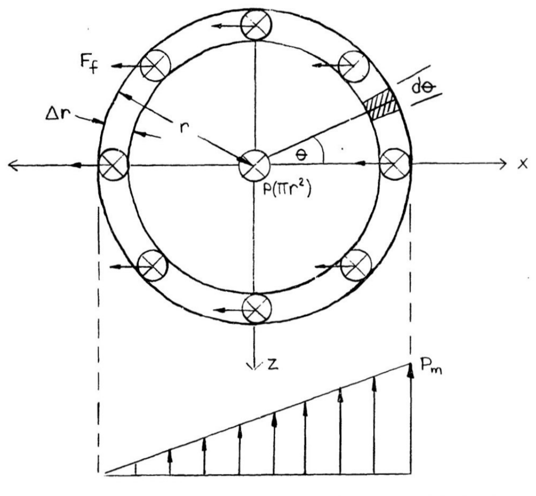


Figure 4: Pressure Distribution on Incremental Area Element of Disc

3.3.2 Equilibrium Moment Balance

Moments taken about a specified point must also be balanced to satisfy the conditions of static equilibrium. Moments will be taken about the point directly opposite the point of leakage. All forces in the x and y directions generate moments about this point having magnitude equal to the product of the applied load and the distance between the load and the axis about which the moments are taken. Individual moment components can therefore be calculated directly once the magnitude of the force components and the dimensions of the disc and disc holder are known. The complete moment balance equation is expressed as follows:

$$\frac{\Pi r^2 \Delta r P_m}{2} + \Pi r^3 P + Fhsin\ \Phi = F(r-c)cos\ \Phi \tag{5}$$

Appendix B can be referenced for a more detailed derivation and explanation of this equation.

3.3.3 Calculation of Leak Pressure

Equations 4 and 5 can be solved for the two unknown variables, P and P_m. Solving for P_m gives the maximum pressure acting on the sealing surface of the disc as follows:

$$P_m = \frac{Fcos\ \Theta}{\Pi r \Delta r} - \frac{Pr}{\Delta r} \tag{6}$$

This value becomes a concern only when it reaches a level where the disc sealing surface may actually be damaged or deformed from the loading.

Solving for P will indicate the system pressure at which the valve starts to leak, given the specified loading conditions, set pressure, and design parameters. This leak pressure is given by the following expression:

$$P = 2F\frac{[r/2 \cdot cos\ \Phi - c \cdot cos\ \Phi - h \cdot sin\ \Phi]}{\Pi r^3} \tag{7}$$

The results of this model become more meaningful when the leak pressure, P is expressed as a fraction of the set pressure, P_{set}. This ratio is given simply as follows:

$$\frac{P}{P_{set}} = \frac{P}{F/\Pi r^2} \tag{8}$$

This ratio will more clearly indicate where in its specified operating range the valve will begin to leak.

4 Analysis and Implementation of Model

4.1 Allowable Levels of Angular and Misaligned Loading

The model for seat leakage indicates the pressure at which a valve will first leak under a specified set of design parameters and loading conditions. By varying the loading conditions for a given set of design parameters, the tolerable limits of angular and misaligned loading can be determined. The model is able to address the individual or combined effects of these loading conditions. It is important to note that the limits established by the model are the limits at which no leakage will occur at 90% of set pressure. The model, therefore errs slightly on the safe side, since API Standard 527 does specify a certain allowable leakage rate depending on the size of the valve.

A DN 20 inlet by DN 25 outlet Crosby JMBL pressure relief valve will be used as a base case to illustrate the development of tolerable limits for angular and misaligned loading. The disc radius and distance from the point of load application to the sealing surface of the disc are specified for the JMBL model. The set pressure of the JMBL valve can range anywhere from 0.7 bar to 345 bar. For this study, the set pressure will be specified at 0.7 bar. This specification also dictates the magnitude of the applied spring load, F, since the relationship between set pressure and applied load is as follows:

$$P_{set} = \frac{F}{\Pi r^2} \tag{9}$$

Table 4-1 summarizes the specified parameters for the JMBL valve to be evaluated:

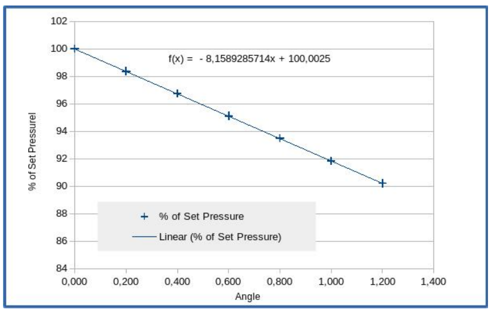
Table 1: Specified Parameters for Crosby JMBL Valve

Symbol	Parameter	Specified Value
h	dist. from applied load	2.0 centimeters
r	disc radius	0.8 centimeters
F	applied load	155.0 Newton
P_{set}	set pressure	0.7 bar

With these parameters held constant, the angle of the applied load, Φ, and the misalignment of the applied load, c, can be varied from zero to the value that produces initial leakage at 90% set pressure. In a case of ideal loading (Φ = 0, c = 0) under this model, leakage will occur only at 100% set pressure when the valve opens completely.

Table 2: Leak Pressures for Angular Loading

Loading Angle Φ (degrees)	Leak Pressure P (psi)	% of Set Pressure (psi)
0.0	100	100
0.2	98.37	98.37
0.4	96.74	96.74
0.6	95.11	95.11
0.8	93,48	93.48
1.0	91.84	91.84
1.2	90.21	90.21
1.225	90.00	90.00



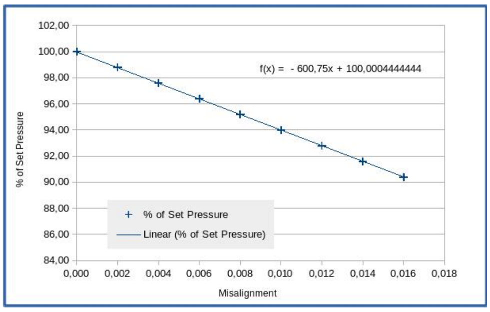
The allowable level of angular loading for the specified Crosby JMBL valve is given in Table 4-2. The misalignment is specified to be zero in order to isolate the effects of angular loading.

It can be seen from Table 4-2 that in the presence of an angular load with no misalignment, an angle of 1.225 degrees can be tolerated with no leakage up to 90% of set pressure.

The allowable level of misalignment for the specified Crosby JMBL valve is given in Table 4-3 which follows. The angle of the applied load is specified to be zero in order to isolate the effects of misaligned loading.

Table 3: Leak Pressures for Misaligned Loading

Misalignment c (inches)	Leak Pressure P (psi)	% of Set Pressure (psi)
0.000	100	100
0.002	98.80	98.80
0.004	97.60	97.60
0.006	96.39	96.39
0.008	95.20	95.20
0.010	93.99	93.99
0.012	92.79	92.79
0.014	91.59	91.59
0.016	90.39	90.39
0.0167	90.00	90.00



It can be seen from Table 4-3 that in the presence of a misaligned load with no angular loading, a misalignment of 0.0167 inches can be tolerated with no leakage up to 90% of set pressure. The combined effects of angular and misapplied loading can also be evaluated by this model to obtain the tolerable limits of any combination of these loading conditions.

4.2 Optimization of Design Parameters

The mathematical model for valve leakage can be used to specify the tolerable limits of misapplied loading for any set of specified design parameters. Relevant geometric parameters can be varied around the base case values of the JMBL valve to identify performance trends and optimal values for these parameters. Variations in the disc radius, r, and the distance from the point of load application to the sealing surface of the disc, h, can significantly affect the valve response to conditions of misaligned and angular loading. These two parameters will be assigned values 10%, 25%, and 50% above and below the base values for the JMBL valve.

These experimental values for h, when substituted in for the base value of 0.777 inches, yield the following limits for angular and misaligned loading. It is important to note that the effects of these loading conditions are treated independently.

Table 4 indicates that smaller distances between the point of load application and the sealing surface can reduce the valve leakage under a given set of loading conditions. Decreasing the base value by 50% nearly doubles the tolerable angle of load application.

These results also seem to indicate that the tolerable misalignment is independent of the value of h. This

Table 4: Limits of Misapplied Loading For Varied Values of h

Distance to applied load h (inches)	Tolerable Angle Φ (degrees)	Tolerable Misalingment c (inches)
0.389	2.435	0.0167
0.583	1.630	0.0167
0.699	1.363	0.0167
0.777	1.225	0.0167
0.855	1.115	0.0167
0.971	0.981	0.0167
1.165	0.818	0.0167

is true only in the case where the angle of applied load is zero.

The experimental values for disc radius, when substituted for the base value of 0.333 inches, yield the following limits for angular and misaligned loading. The applied load, F, is varied with the radius to maintain a constant set pressure of 100 psi.

Table 5: Limits of Misapplied Loading for Varying Disc Radii

Disc Radius r (inches)	Applied Load F (lbs)	Tolerable Angle Φ (degrees)	Tolerable Misalignment c (inches)
0.167	8.76	0.615	0.0084
0.250	19.64	0.920	0.0125
0.300	28.27	1.105	0.0150
0.333	34.84	1.225	0.0167
0.366	42.08	1.345	0.0183
0.416	54.37	1.530	0.0208
0.500	78.54	1.835	0.0250

Table 4-5 indicates that a larger disc radius will reduce the valve leakage under a given set of loading conditions. The tolerable limits of angular and misaligned loading are both expanded with an increased radius.

4.3 Performance Concerns and Recommendations

With leakage as the major concern, this model would suggest increasing the disc radius and minimizing the distance between the point of load application and the disc sealing surface. There are, however, other considerations that must be included in a more complete evaluation of valve performance. Implementation of these modifications to valve components must not introduce new problems or sacrifice overall valve performance for reduced leakage.

The distance between the point of load application and the disc sealing surface, h, should be minimized in an effort to reduce leakage. However it is also important to maintain a sufficiently long guiding surface between the disc holder and the guide. This guiding surface is essential to help maintain correct alignment and ensure smooth travel of the disc holder. Every effort should be made to maintain the length of the guiding surface while minimizing h. Figure 4-1 clearly illustrates the relevant dimensions of the disc holder. By increasing the depth of the bore in the top surface of the disc holder, this performance tradeoff can be avoided. The 3/4 inch inlet by 1 inch outlet JMBL valve is currently manufactured with a 9/32 inch depth bore. Increasing the depth of this bore by as little as 2/32 inch will expand the tolerable limit of angular loading by nearly 10%. I would recommend manufacturing and testing several prototypes incorporating a deeper bore.

There is also a performance tradeoff that results from increasing the disc radius. Implementing a larger disc radius requires the valve to be housed in a larger external cylinder. This increased cavity size reduces the pressure buildup in the cavity above the disc, and may therefore induce flutter during pressure release. It is therefore necessary to perform sufficient prototype testing to ensure that any increase in disc radius will not result in flutter.

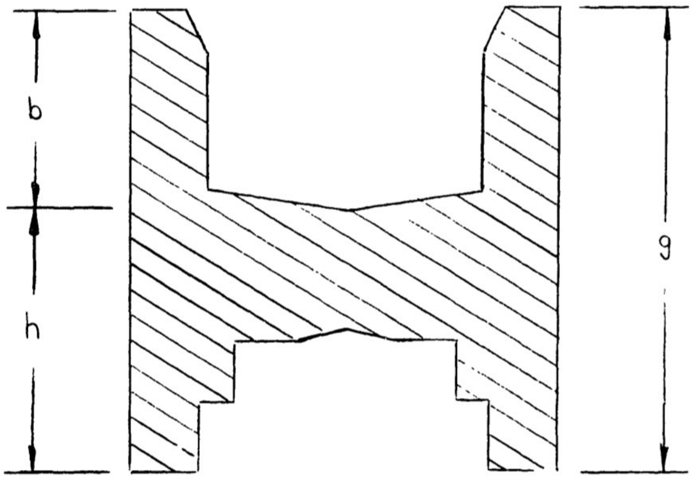


Figure 5: Pressure Release Valve Disc Holder

5 Conclusions

A mathematical model of pressure relief valve seat leakage has been developed to evaluate the effects of misapplied loading on valve leakage. This model incorporates relevant loading conditions as well as geometric properties of valve components to establish tolerable limits of angular and misaligned loading. The effects of these misapplied loading conditions can be determined independently or in combination for any specified set of valve design parameters.

The model also indicates design modifications that will minimize the effects of misaligned and offcentered loading. Implementation of this model and optimization data can save Crosby time in design, production, and testing of valve prototypes by providing a theoretical basis for valve component designs which minimize leakage problems.

6 Appendix A
Equilibrium Force Balance Equations

forces in x direction: The component of the applied load, applied at an angle ϕ, in the x direction is given by the following expresion:

$$F \cdot sin\ \phi$$

The frictional force between the sealing surfaces of the disc and noozle, F_f, is balanced with the x-component of the applied load to yield the following force balance equation:

$$F \cdot sin\ \phi = F_f$$

forces in y direction: The component of the applied load, applied at an angle ϕ, in the y direction is given by the following expression

$$F \cdot cos\ \phi$$

The resultant force of the system pressure acting over the area of the disc insert is expressed as the product of the system pressure and the disc area:

$$\Pi \cdot r^2 \cdot P$$

At the point of leak initiation along the sealing surface of the disc varies from zero at the location of leakage to a maximum value, P_m, directly opposite this point. Since this pressure force varies with position on the disc, a differential force element must be integrated around the entire disc:

$$2 \int \frac{r + cos\ \Theta}{2r} P_m (r \cdot \Delta r) d\Theta$$

Evaluating this integral yields the following expression:

$$\Pi r \Delta r P_m$$

These three force components can be balanced to yield the following equilibrium force balance equation in the x direction:

$$F \cdot cos\ \Phi = \Pi r \Delta r P_m + \Pi r^2 P$$

7 Appendix B
Equilibrium Moment Balance Equations

The y component of the applied load generates a moment about the z-axis which resists lifting and maintains the pressure seal. The magnitude of this moment component is expressed as follows:

$$(F \cdot cos\ \Phi)(r - c)$$

where $(r - c)$ is the perpendicular distance between the point of load application an the perimeter of the sealing surface. All other moment components about the z-axis act to oppose this moment and lift the disc. The x component of the applied load generates a moment of the following magnitude:

$$(F \cdot sin\ \Phi)(h)$$

where h is the perpendicular distance from the point of load application to the sealing surface. The resultant force of the system pressure generates a moment given by the following expression:

$$(\Pi r^2 P)(r)$$

The distributed pressure force along the sealing surface of the disc generates a moment distribution that must be integrated around the circumference of the disc.

$$2\int_0^{\pi} \frac{r + cos\ \Theta}{2r} P_m (r\Delta r)(r - rcos\ \Theta) d\Theta$$

Evaluating this integral yields the following expression:

$$\frac{\Pi r^2 \Delta r P_m}{2}$$

These individual moment components can be combined to yield the following equilibrium moment balance equation about the z-axis:

$$\frac{\Pi r^2 \Delta r P_m}{2} + \Pi r^3 P + Fhsin\ \Phi = F(r - c)cos\ \Phi$$